X-ray tomography and discretization of inverse problems

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In this talk we consider the question how inverse problems posed for continuous objects, for instance for continuous functions, can be discretized. This means the approximation of the problem by infinite dimensional inverse problems. We will consider linear inverse problems of the form $m = Af + \varepsilon$. Here, the function m is the measurement, A is a ill-conditioned linear operator, u is an unknown function, and ε is random noise. The inverse problem means determination of u when m is given. In particular, we consider the X-ray tomography with sparse or limited angle measurements where A corresponds to integrals of the attenuation function $u(x)$ over lines in a family γ. The traditional solutions for the problem include the generalized Tikhonov regularization and the estimation of u using Bayesian methods. To solve the problem in practice u and m are discretized, that is, approximated by vectors in an infinite dimensional vector space. We show positive results when this approximation can successfully be done and consider examples of problems that can appear. As an example, we consider the total variation (TV) and Besov norm penalty regularization, the Bayesian analysis based on total variation prior and Besov priors.